

Hong Kong Mathematics Olympiad (2000 – 2001)

Final Event 1 (Group)

香港数学竞赛 (2000 – 2001)

决赛项目 1 (团体)

除非特别声明，答案须用数字表达，并化至最简。

Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

1. 已知 $(a+b+c)^2 = 3(a^2+b^2+c^2)$ 及 $a+b+c=12$ ，求 a 的值。

Given that $(a+b+c)^2 = 3(a^2+b^2+c^2)$ and $a+b+c=12$, find the value of a .

2. 已知 $b\left[\frac{1}{1 \times 3} + \frac{1}{3 \times 5} + \cdots + \frac{1}{1999 \times 2001}\right] = 2 \times \left[\frac{1^2}{1 \times 3} + \frac{2^2}{3 \times 5} + \cdots + \frac{1000^2}{1999 \times 2001}\right]$ ，求 b 的值。

Given that $b\left[\frac{1}{1 \times 3} + \frac{1}{3 \times 5} + \cdots + \frac{1}{1999 \times 2001}\right] = 2 \times \left[\frac{1^2}{1 \times 3} + \frac{2^2}{3 \times 5} + \cdots + \frac{1000^2}{1999 \times 2001}\right]$, find the value of b .

3. 一六位数 $1234xy$ 能同时被 8 和 9 整除。已知 $x+y=c$ ，求 c 的值。

A six-digit number $1234xy$ is divisible by both 8 and 9. Given that $x+y=c$, find the value of c .

4. 已知 $\log_x t = 6$ ， $\log_y t = 10$ ， $\log_z t = 15$ 。若 $\log_{xyz} t = d$ ，求 d 的值。

Suppose $\log_x t = 6$, $\log_y t = 10$ and $\log_z t = 15$. If $\log_{xyz} t = d$, find the value of d .

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Final Event 2 (Group)

香港数学竞赛 (2000 – 2001)

决赛项目 2 (团体)

除非特别声明，答案须用数字表达，并化至最简。

Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

1. 已知 $x = \sqrt{7-4\sqrt{3}}$ 及 $\frac{x^2-4x+5}{x^2-4x+3} = a$ ，求 a 的值。

Given that $x = \sqrt{7-4\sqrt{3}}$ and $\frac{x^2-4x+5}{x^2-4x+3} = a$, find the value of a .

2. E 是长方形 $ABCD$ 内一点。已知 EA 、 EB 、 EC 和 ED 的长度分别为 2 、 $\sqrt{11}$ 、 4 及 b ，求 b 的值。

E is an interior point of the rectangle $ABCD$. Given that the lengths of EA , EB , EC and ED are 2 , $\sqrt{11}$, 4 and b respectively, find the value of b .

3. 已知 $111111222222 = c \times (c+1)$ ，求 c 的值。

Given that $111111222222 = c \times (c+1)$, find the value of c .

4. 已知 $\cos 16^\circ = \sin 14^\circ + \sin d^\circ$ and $0 < d < 90$ ，求 d 的值。

Given that $\cos 16^\circ = \sin 14^\circ + \sin d^\circ$ and $0 < d < 90$, find the value of d .

Hong Kong Mathematics Olympiad (2000 – 2001)

Final Event 3 (Group)

香港数学竞赛 (2000 – 2001)

决赛项目 3 (团体)

除非特别声明，答案须用数字表达，并化至最简。

Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

1. 已知方程 $\sqrt{3x+1} + \sqrt{3x+6} = \sqrt{4x-2} + \sqrt{4x+3}$ 的解为 a ，求 a 的值。

Given that the solution of the equation $\sqrt{3x+1} + \sqrt{3x+6} = \sqrt{4x-2} + \sqrt{4x+3}$ is a , find the value of a .

2. 已知方程 $x^2y - x^2 - 3y - 14 = 0$ 只得一组正整数解 (x_0, y_0) 。若 $x_0 + y_0 = b$ ，求 b 的值。

Suppose the equation $x^2y - x^2 - 3y - 14 = 0$ has only one positive integral solution (x_0, y_0) . If $x_0 + y_0 = b$, find the value of b .

3. $ABCD$ 是一圆内接四边形。 AC 和 BD 相交于 G 。已知 $AC = 16$ cm, $BC = CD = 8$ cm, $BG = x$ cm 和 $GD = y$ cm。若 x 和 y 皆为整数且 $x + y = c$ ，求 c 的值。

$ABCD$ is a cyclic quadrilateral. AC and BD intersect at G . Suppose $AC = 16$ cm, $BC = CD = 8$ cm, $BG = x$ cm and $GD = y$ cm. If x and y are integers and $x + y = c$, find the value of c .

4. 已知 $5^{\log 30} \times \left(\frac{1}{3}\right)^{\log 0.5} = d$ ，求 d 的值。

Given that $5^{\log 30} \times \left(\frac{1}{3}\right)^{\log 0.5} = d$, find the value of d .

Hong Kong Mathematics Olympiad (2000 – 2001)

Final Event 4 (Group)

香港数学竞赛 (2000 – 2001)

决赛项目 4 (团体)

除非特别声明，答案须用数字表达，并化至最简。

Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

1. $x_1 = 2001$ 。当 $n > 1$, $x_n = \frac{n}{x_{n-1}}$ 。已知 $x_1 x_2 x_3 \cdots x_{10} = a$, 求 a 的值。

$x_1 = 2001$. When $n > 1$, $x_n = \frac{n}{x_{n-1}}$. Given that $x_1 x_2 x_3 \cdots x_{10} = a$, find the value of a .

2. 已知 $1^3 + 2^3 + 3^3 + \cdots + 2001^3$ 的个位数字为 b , 求 b 的值。

Given that the unit digit of $1^3 + 2^3 + 3^3 + \cdots + 2001^3$ is b , find the value of b .

3. 甲、乙两人在一圆形跑道上同时同地相背以均速开跑。他们第一次相遇后，乙再跑 1 分钟到达原起步点。已知甲和乙分别需要 6 分钟和 c 分钟绕跑道一周，求 c 的值。

A and B ran round a circular path with constant speeds. They started from the same place and at the same time in opposite directions. After their first meeting, B took 1 minute to go back to the starting place. If A and B need 6 minutes and c minutes respectively to complete one round of the path, find the value of c .

4. 方程 $x^2 - 45x + m = 0$ 的两个根皆为质数。已知两根的平方和为 d , 求 d 的值。

The roots of the equation $x^2 - 45x + m = 0$ are prime numbers. Given that the sum of the squares of the roots is d , find the value of d .